which are known, i.e., by a bridge method [5]. Thus, a new method has been developed [4] for determining the group of thermophysical properties which can be classified as a differential-bridge thermometric method, the calculation formulas for which are given by equations (12), (15), and (16).

NOTATION

x, τ, local values of the spatial coordinate and time; H, half-thickness of plate; T, temperature; T_{0_1}, T_{0_2}, initial constant temperatures of surfaces; b, rate of change of temperature; q, heat flux density; q_0, initial heat flux density through surface of plate whose coordinate is $x = H$; q_1, q_2, heat flux densities at cross sections $x = x_1$, $x = x_2$; λ, thermal conductivity; a, temperature conductivity; $X = x/H$, dimensionless coordinate; $Fo = a\tau/x^2$, Fourier number; $c\rho$, volumetric heat capacity, $c\rho = \lambda/a$; δT, δq, temperature and heat flux corrections; κ, γ, relative temperature and heat flux corrections; R, thermal resistance.

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METHOD FOR DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF ORTHOTROPIC BODIES

Yu. M. Kolyano

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The solution for the inverse coefficient problem of heat conduction for an orthotropic body is proposed.

There exists a nondestructive monitoring method for determining the coefficients of thermal conductivity of orthotropic bodies [1]. In the method a standard sample (isotropic half-space), whose thermal conductivity is known, is heated together with the sample of interest, placed in series, with a mobile point source of heat moved along the surface of the samples at a constant rate and the excess limiting temperature of the surface of the samples along the line of heating is measured with the help of a temperature sensor moved at the same rate as the source at a fixed distance from it. The sample under study is made with two mutually perpendicular flat surfaces, perpendicular to its principal axis of heat conduction, and scanning over the flat surfaces along each of the three principal axes of heat conduction is performed in sequence. The coefficient of thermal diffusivity and the volume heat capacity are not determined

To determine the complex of thermophysical characteristics of orthotropic bodies by the method of nondestructive monitoring, we shall examine three samples in the form of orthotropic half-spaces $z \geq 0$, $x \geq 0$, $y \geq 0$, over whose surfaces $z = 0$, $x = 0$, and $y = 0$ a source of heat with power q [W] and a sensor for measuring the temperature at a fixed distance ℓ for the heat source (Fig. 1) move with a constant velocity v in the positive direction along

Institute of Applied Problems in Mechanics and Mathematics, Academy of Sciences of the Ukrainian SSR, Lvov.

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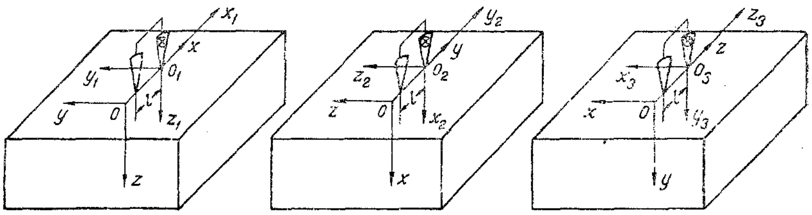

Fig. 1. Geometric model of the method for determining the thermophysical characteristics of orthotropic bodies.

the x, y, and z axes. Temperature ranges in which the thermophysical characteristics are temperature-independent are studied. The initial temperature equals t_i. The temperature of the body at infinity equals t_i, and its derivatives with respect to the coordinates equal zero.

In this case, to determine the excess temperature in the first sample we have the heat-conduction equation

$$\frac{\partial^2\Theta}{\partial X^2}+\frac{\partial^2\Theta}{\partial Y^2}+\frac{\partial^2\Theta}{\partial Z^2}=c_v\dot{\Theta} \tag{1}$$

and the boundary conditions

$$\Theta|_{z\to\infty}=0,\quad \left.\frac{\partial\Theta}{\partial Z}\right|_{z=0}=-Q\delta(X-V_1\tau)\,\delta(Y)\,S_+(\tau), \tag{2}$$

$$\Theta|_{\tau=0}=0,\quad \left.\frac{\partial\Theta}{\partial X}\right|_{|x|\to\infty}=0,\quad \left.\frac{\partial\Theta}{\partial Y}\right|_{|y|\to\infty}=0, \tag{3}$$

where $\Theta = t - t_i$, and t is the temperature distribution; $\delta(\xi)$ is the Dirac delta function; $X = x/\sqrt{\lambda_1}$, $Y = y/\sqrt{\lambda_2}$, $Z = z/\sqrt{\lambda_3}$, $Q = q/\sqrt{\lambda_1\lambda_2\lambda_3}$, $V_1 = v/\sqrt{\lambda_1}$, $\dot{\Theta} = \partial\Theta/\partial\tau$, and τ is the time.

We Fourier transform Eq. (1) and the boundary conditions (2) under the conditions (3) with respect to X and Y and Laplace transform with respect to τ. This gives

$$\frac{d^2\bar{\Theta}}{dZ^2}=\gamma^2\bar{\Theta}, \tag{4}$$

$$\bar{\Theta}|_{z\to\infty}=0,\quad \left.\frac{d\bar{\Theta}}{dZ}\right|_{z=0}=-\frac{Q}{\sqrt{2\pi}}\,\tilde{b}(\xi,\ s), \tag{5}$$

where

$$\bar{\Theta}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\Theta\exp[i(\xi X+\eta Y)-s\tau]\,dXdYd\tau,\quad \gamma=\sqrt{\xi^2+\eta^2+c_v s},$$

$$\tilde{b}(\xi,\ s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{0}^{\infty}\delta(X-V_1\tau)\exp(i\xi X-s\tau)\,dXd\tau,$$

ξ, η, x are the parameters of the Fourier transformations with respect to X and Y and the Laplace transformation with respect to τ

The solution of Eq. (4) with the boundary conditions (5) is written in the form

$$\bar{\Theta}=\frac{Q}{\sqrt{2\pi}\gamma}\,\tilde{b}(\xi,\ s)\exp(-\gamma Z). \tag{6}$$

We perform the inverse Fourier—Laplace transformation in (6) using the convolution theorem for the Fourier and Laplace transformations and the handbook data in [2]. As a result we obtain the following expression for the excess temperature:

$$\Theta=\frac{Q}{4\pi R_1}\exp\left(-\frac{X_1V_1c_v}{2}\right)\left[\exp\left(\frac{R_1}{2}V_1c_v\right)\operatorname{erfc}\left(\frac{R_1}{2}\sqrt{\frac{c_v}{\tau}}+\right.\right.$$
$$\left.\left.+\frac{V_1}{2}\sqrt{\tau c_v}\right)+\exp\left(-\frac{R_1V_1c_v}{2}\right)\operatorname{erfc}\left(\frac{R_1}{2}\sqrt{\frac{c_v}{\tau}}-\frac{V_1}{2}\sqrt{\tau c_v}\right)\right], \tag{7}$$

where $R_1 = \sqrt{X_1^2 + Y_1^2 + Z_1^2}$; $X_1 = X - V_1\tau$; $\operatorname{erfc}\zeta = 1 - \operatorname{erf}\zeta$; $\operatorname{erf}\zeta$ is the error function; $Y_1 = y_1\sqrt{\lambda_2}$, $Z_1 = z_1/\sqrt{\lambda_3}$, and, $0_1x_1y_1z_1$ is a moving coordinate system.

For $Z_1 = 0$, $Y_1 = 0$ we have

$$\Theta|_{\substack{Z_1=0,\\ Y_1=0}} = \frac{Q}{4\pi|X_1|}\exp\left(-\frac{X_1}{2}V_1c_v\right)\left[\exp\left(\frac{|X_1|}{2}V_1c_v\right)\times \right.$$
$$\left.\times\operatorname{erfc}\left(\frac{|X_1|}{2}\sqrt{\frac{c_v}{\tau}}+\frac{V_1}{2}\sqrt{c_v\tau}\right)+\exp\left(-\frac{|X_1|}{2}V_1c_v\right)\operatorname{erfc}\left(\frac{|X_1|}{2}\sqrt{\frac{c_v}{\tau}}-\frac{V_1}{2}\sqrt{c_v\tau}\right)\right]. \tag{8}$$

In the quasistationary state ($\tau \to \infty$) from (8) it follows that

$$\Theta|_{\substack{Z_1=0,\,Y_1=0,\\ \tau\to\infty}} = \frac{Q}{2\pi|X_1|}\exp\left(-\frac{X_1+|X_1|}{2}V_1c_v\right). \tag{9}$$

Let $X_1 = -L_1 = -\ell/\sqrt{\lambda_1}$. Then it follows from (9) that the measured temperature is given by the relation

$$\Theta_1 = \frac{Q}{2\pi L_1} = \frac{q}{2\pi\sqrt{\lambda_2\lambda_3}l}. \tag{10}$$

The excess temperature with the energy source and the temperature sensor moving along the y axis in the x = 0 plane of the half-space $x \geq 0$ can be determined analogously. It has the form

$$\Theta = \frac{Q}{4\pi R_2}\exp\left(-\frac{Y_2}{2}V_2c_v\right)\left[\exp\left(\frac{R_2}{2}V_2c_v\right)\operatorname{erfc}\left(\frac{R_2}{2}\sqrt{\frac{c_v}{\tau}}+\right.\right.$$
$$\left.\left.+\frac{V_2}{2}\sqrt{c_v\tau}\right)+\exp\left(-\frac{R_2}{2}V_2c_v\right)\operatorname{erfc}\left(\frac{R_2}{2}\sqrt{\frac{c_v}{\tau}}-\frac{V_2}{2}\sqrt{c_v\tau}\right)\right], \tag{11}$$

where $Y_2 = Y - V_2\tau$, $R_2 = \sqrt{X_2^2 + Y_2^2 + Z_2^2}$, $V_2 = v/\sqrt{\lambda_2}$, $X_2 = x_2/\sqrt{\lambda_1}$, $Z_2 = z_2/\sqrt{\lambda_3}$, and $0_2x_2y_2z_2$ is a moving coordinate system

For $X_2 = 0$, $Z_2 = 0$, and $\tau \to \infty$ we have

$$\Theta|_{\substack{X_2=0,\,Z_2=0,\\ \tau\to\infty}} = \frac{Q}{2\pi|Y_2|}\exp\left(-\frac{Y_2+|Y_2|}{2}V_2c_v\right). \tag{12}$$

Let $Y_2 = -L_2 = -\ell/\sqrt{\lambda_2}$. Then from (12) it follows that the measured temperature is determined by the relation

$$\Theta_2 = \frac{Q}{2\pi L_2} = \frac{q}{2\pi\sqrt{\lambda_1\lambda_3}l}. \tag{13}$$

If the energy source and the temperature sensor are moved with a constant velocity v along the z axis over the surface y = 0 of the half-space $y \geq 0$, then we find analogously

$$\Theta = \frac{Q}{4\pi R_3}\exp\left(-\frac{Z_3}{2}V_3c_v\right)\left[\exp\left(\frac{R_3}{2}V_3c_v\right)\operatorname{erfc}\left(\frac{R_3}{2}\sqrt{\frac{c_v}{\tau}}+\right.\right.$$
$$\left.\left.+\frac{V_3}{2}\sqrt{c_v\tau}\right)+\exp\left(-\frac{R_3}{2}V_3c_v\right)\operatorname{erfc}\left(\frac{R_3}{2}\sqrt{\frac{c_v}{\tau}}-\frac{V_3}{2}\sqrt{c_v\tau}\right)\right], \tag{14}$$

where $Z_3 = Z - V_3\tau$; $R_3 = \sqrt{X_3^2 + Y_3^2 + Z_3^2}$; $X_3 = x_3/\sqrt{\lambda_1}$; $Y_3 = y_3/\sqrt{\lambda_3}$; and $0_3x_3y_3z_3$ is a moving coordinate system.

For $X_3 = 0$, $Y_3 = 0$ and $\tau \to \infty$ we have

$$\Theta|_{\substack{X_3=0,\,Y_3=0,\\ \tau\to\infty}} = \frac{Q}{2\pi|Z_3|}\exp\left(-\frac{Z_3+|Z_3|}{2}V_3c_v\right). \tag{15}$$

If $Z_3 = -L_3 = -\ell/\sqrt{\lambda_3}$, then from (15) we find that the measured temperature is given by the relation

$$\Theta_3 = \frac{Q}{2\pi L_3} = \frac{q}{2\pi\sqrt{\lambda_1\lambda_2}l}. \tag{16}$$

Since the temperature sensor is located at a distance ℓ from the heat source with power

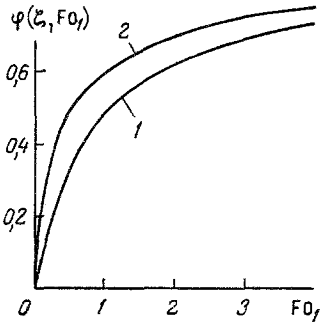

Fig. 2. Function $\varphi(\zeta, Fo_1)$ as a function of the Fourier criterion Fo_1 for $\zeta = 0.05$ (curve 1) and $\zeta = 0.5$ (curve 2).

q in each of the three samples, the quantity $q/2\pi\ell = \omega$ remains constant during the heating process and the temperature measurements. From the expressions (10), (13), and (16) we determine the coefficients of thermal conductivity along the x, y, and z axes in the form

$$\lambda_1 = \frac{\Theta_1\omega}{\Theta_2\Theta_3},\ \lambda_2 = \frac{\Theta_2\omega}{\Theta_1\Theta_3},\ \lambda_3 = \frac{\Theta_3\omega}{\Theta_1\Theta_2}. \tag{17}$$

From the expression (8) with $X_1 = -L_1$ at time $\tau = \tau_0$ we have

$$\Theta_0 = \Theta\Big|_{\substack{Y=0,\ Z=0,\\ X_1=-L_1,\ \tau=\tau_0}} = \frac{\Theta_1}{2}\left[\exp\left(\frac{\zeta}{Fo_1}\right)\operatorname{erfc}\left(\frac{1+\zeta}{2\sqrt{Fo_1}}\right) + \operatorname{erfc}\left(\frac{1-\zeta}{2\sqrt{Fo_1}}\right)\right] = \Theta_1\varphi(\zeta,\ Fo_1), \tag{18}$$

where $Fo_1 = a_1\tau_0/\ell^2$; $\zeta = v\tau_0/\ell$; $a_1 = \lambda_1/c_V$.

Figure 2 shows a graph of the function $\varphi(\zeta, Fo_1)$ (18) for $\zeta = 0.05$; 0.5 as a function of Fo_1. Knowing the measured quantity $\vartheta = \Theta_0/\Theta_1$, we determined from the graph the corresponding value of Fo_1. Having determined Fo_1, from the formula

$$a_1 = \frac{Fo_1}{\tau_0} l^2 \tag{19}$$

we find the coefficient of thermal diffusivity along the x axis. If the coefficient of thermal diffusivity a_1 is known, then the volume heat capacity can be found from the formula

$$c_v = \lambda_1/a_1. \tag{20}$$

If the volume heat capacity c_V (20) and the coefficients of thermal conductivity λ_2 and λ_3 have been determined, then the coefficients of thermal diffusivity along the y and z axes can be determined from the formulas

$$a_2 = \lambda_2/c_v,\ a_3 = \lambda_3/c_v. \tag{21}$$

Thus, the entire complex of thermophysical characteristics (17) and (19)-(21) of the orthotropic body has been determined.

The power of the heat source q, appearing in the expressions for the coefficients of thermal conductivity (17) is usually assumed to be given. It can be determined, like in [1], using the corresponding (for example, (10)) result for the isotropic standard semiinfinite sample

$$\Theta_e = \omega/\lambda_e, \tag{22}$$

where λ_e is the known thermal conductivity of the standard sample, Θ_e is its measured excess temperature in the quasistationary thermal state, and $z_1 = 0$, $y_1 = 0$, $x_1 = -\ell$. From here it follows that $q = 2\pi\Theta_e\lambda_e\ell$.

Determining ω from (22) and substituting its value into the expression (17) we arrive at the result obtained in [1], where it is pointed out that the method is characterized by a low error in the temperature measurements and high efficiency. To determine the entire complex of thermophysical characteristics there is no need to insert temperature sensors into the interior volumes of the samples under study.

The proposed method of nondestructive monitoring for investigating the complex of thermophysical characteristics of orthotropic bodies can be used to measure temperatures on lines in the bounding surfaces of the samples.

NOTATION

λ_1, λ_2, λ_3, coefficients of thermal conductivity along the x, y, and z axes; a_1, a_2, and a_3, coefficients of thermal diffusivity along the x, y, and z axes; c_V, volume heat capacity; v, velocity of the heat source and temperature sensor; and q, power of the heat source.

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EFFICIENCY OF "SHIELDLESS" METHOD OF EMPLOYING THE COLD OF VAPORS IN CRYOGENIC VESSELS WITH A WIDE NECK

G. G. Zhun', V. F. Getmanets, R. S. Mikhal'chenko, P. N. Yurchenko, V. A. Miroshnichenko, and V. I. Shalaev

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Methods for raising the efficiency of cryobiological vessels with liquid nitrogen are proposed and realized, and a method for calculating their heat-shielding is developed and tested.

Theoretically [1] the full utilization of the cold of vapors enables reducing by a factor of 1.6-1.7 the flow of heat into small vessels with liquid nitrogen. Most often, for this purpose a large number of radiation shields are placed on the cold neck [2]. This construction is difficult to realize and labor-consuming, its mass is large, and for a number of reasons (small number of shields, decrease in the effective length of the neck, increase in the emissivity of the shields owing to cryogenic deposits, etc.), it does not permit full use of the cold of the vapors. The method of placing heat shields within a packet of screen-vacuum thermal insulation (SVTI), cooled with a drainage pipe [3], is less complicated but less efficient.

In small vessels the full utilization of the cold of vapors can be achieved with the help of a simple "shieldless" method, when all SVTI layers are cooled with the drainage neck, along which they are stretched over the entire length and have a good "thermal" contact with it. The shieldless method is employed in serially produced Kh-34B cryobiological vessels [4], but it has not been adequately tested theoretically and experimentally. This is primarily a result of the fact that there are no experimental data and methods for calculating the components of the heat inflow into the vessel with the indicated construction taking into account the thermal interaction of the SVTI packet and the drainage neck.

The purpose of this work is to test experimental and computational methods for evaluating the components of the heat inflow taking into account the use of the cold of vapors and developing recommendations with regard to their efficiency. The components of the heat inflow (with and without the use of the cold of the vapors) along the neck, its plug, and vapors of the cryogenic component were determined experimentally from their thermal conductivity and the temperature gradient in the lowest cold layers of these elements (5-7 mm thick), found with the help of differential thermocouples. The decrease in the temperature gradient in each element (including also in SVTI) owing to the use of the cold of the vapors determines the efficiency of the "shieldless" method of cooling. The thermal conductivity of the

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